Principles of Communications

EES 351

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4.4 Switching MODEM

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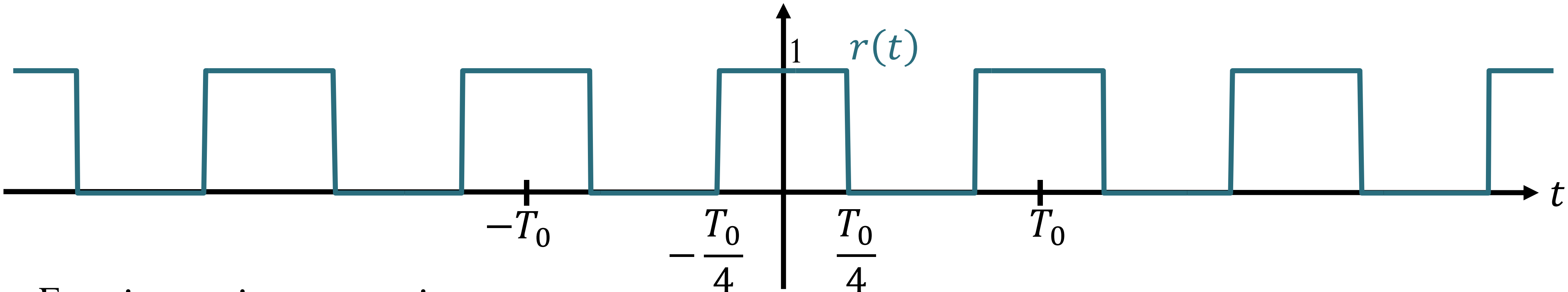
Section 4.4

- **Crucial Skill 4.4.1**: Able to find/draw the output of the switching box both in time and frequency domains.
- Skill 4.4.2: Able to quickly find the output of the switching modulator and switching demodulator.

$$c_k = \frac{1}{2}\operatorname{sinc}\left(k\frac{\pi}{2}\right) = \frac{1}{k\pi}\sin\left(k\frac{\pi}{2}\right)$$

[4.53] Square Wave



Fourier series expansion:

$$r(t) = \frac{1}{2} + \frac{1}{\pi}e^{j(2\pi f_0 t)} - \frac{1}{3\pi}e^{j(2\pi(3f_0)t)} + \frac{1}{5\pi}e^{j(2\pi(5f_0)t)} + \cdots$$

$$+ \frac{1}{\pi}e^{j(2\pi(-f_0)t)} - \frac{1}{3\pi}e^{j(2\pi(-3f_0)t)} + \frac{1}{5\pi}e^{j(2\pi(-5f_0)t)} + \cdots$$

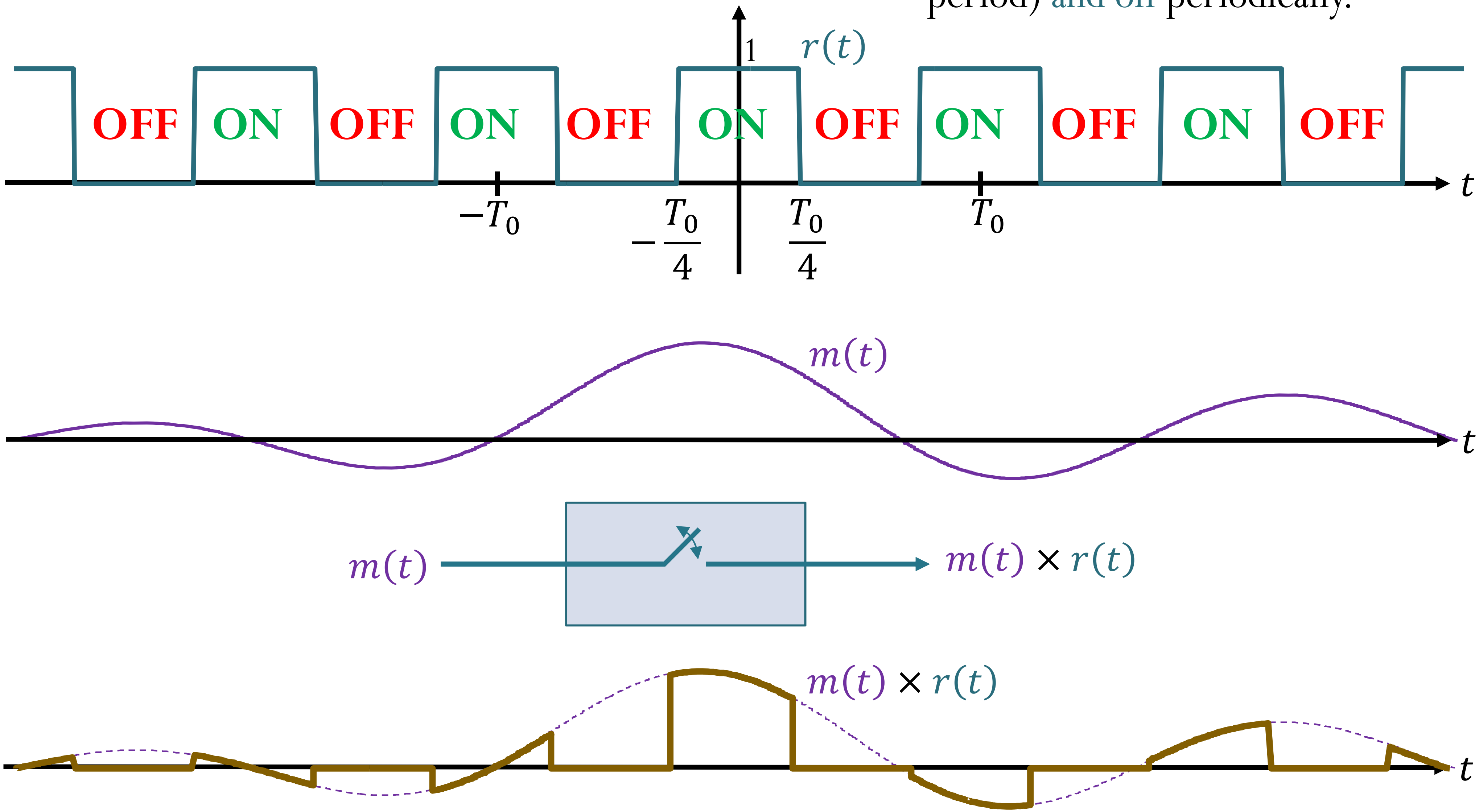
Trigonometric Fourier series expansion:

$$e^{jx} + e^{-jx} = 2\cos(x)$$

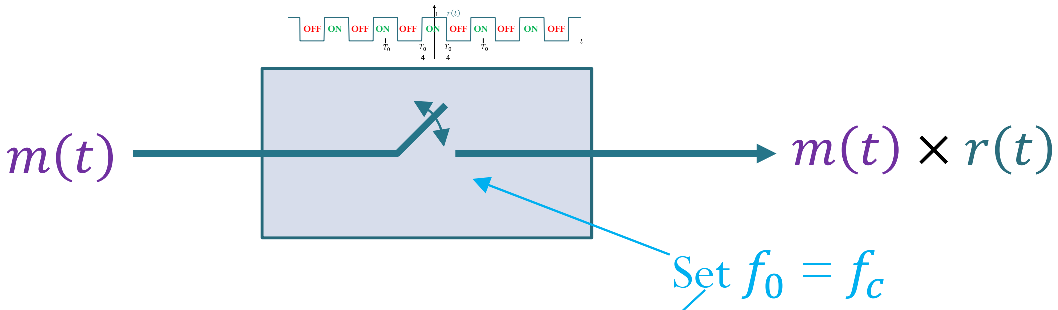
$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_0 t) - \frac{2}{3\pi}\cos(2\pi(3f_0)t) + \frac{2}{5\pi}\cos(2\pi(5f_0)t) + \cdots$$

[4.48a] Switching Operation

Multiplying a signal $m(t)$ by the square-wave $r(t)$ is equivalent to switching $m(t)$ on (for half a period) and off periodically.

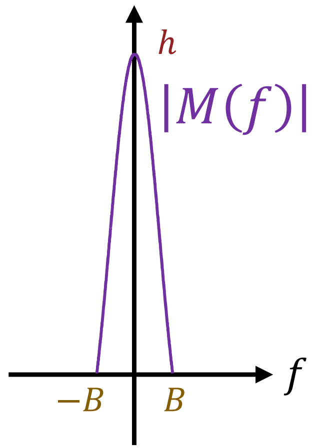


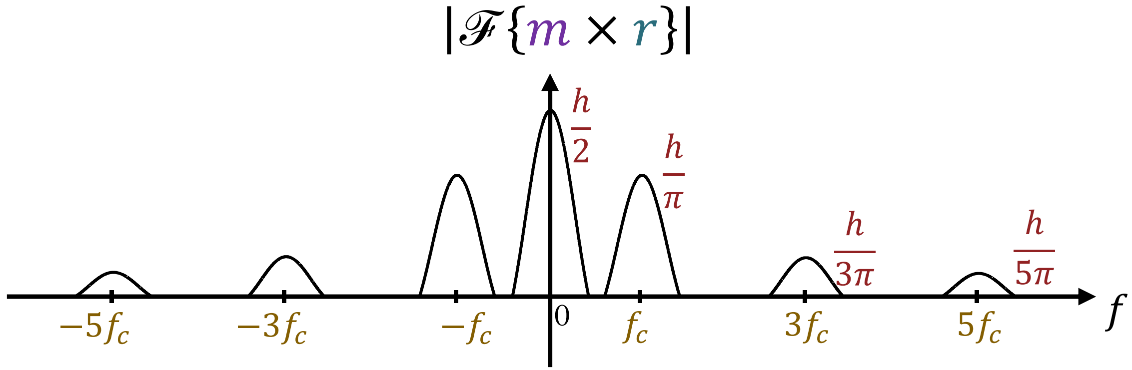
[4.58] Switching Modulator



$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$$m(t)\times r(t) = \frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos(2\pi f_c t) - \frac{2}{3\pi}m(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}m(t)\cos(2\pi(5f_c)t) + \cdots$$

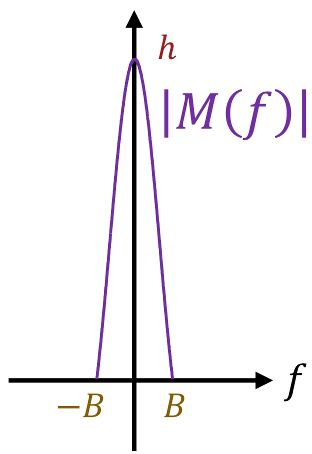


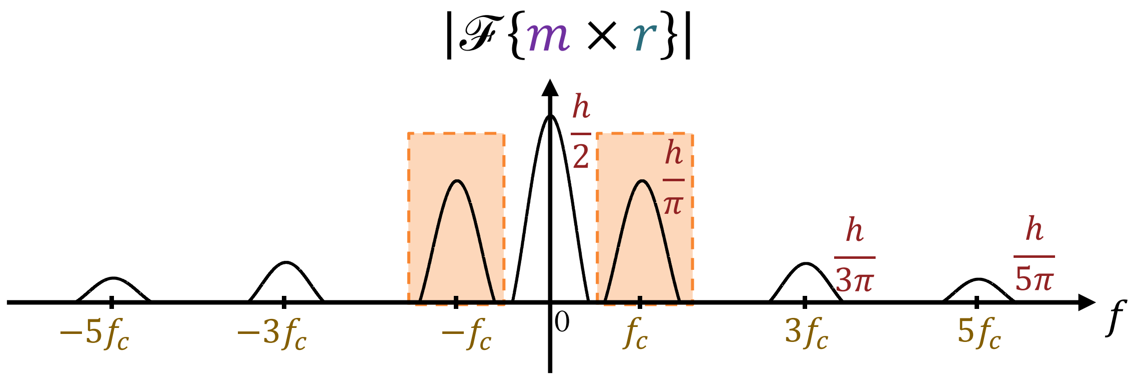


[4.58] Switching Modulator

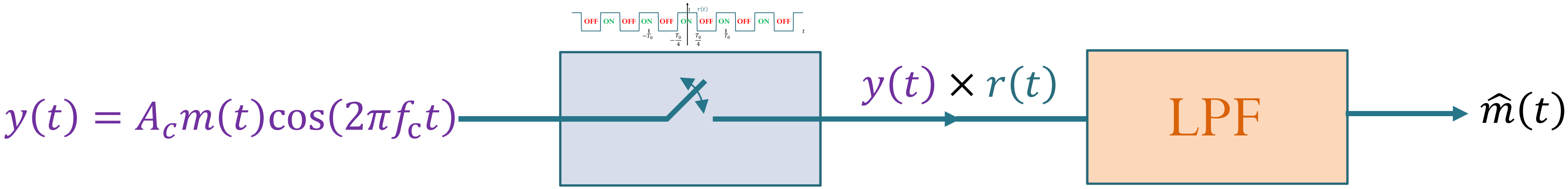
$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$$m(t) \times r(t) = \frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos(2\pi f_c t) - \frac{2}{3\pi}m(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}m(t)\cos(2\pi(5f_c)t) + \cdots$$





[4.59] Switching Demodulator

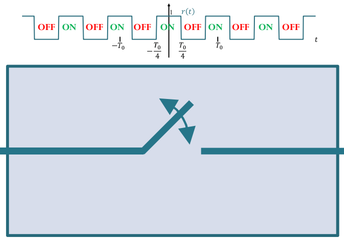


$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$$y(t) \times r(t) = \frac{1}{2}y(t) + \frac{2}{\pi}y(t)\cos(2\pi f_c t) - \frac{2}{3\pi}y(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}y(t)\cos(2\pi(5f_c)t) + \cdots$$

All the calculations in these remaining slides are already included in the lecture note.

[4.59] Switching Demodulator

$y(t) = A_c m(t)\cos(2\pi f_c t)$  $y(t) \times r(t)$

Plug-in

$$y(t)r(t) = \frac{1}{2}y(t) + \frac{2}{\pi}y(t)\cos(2\pi f_c t) - \frac{2}{3\pi}y(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}y(t)\cos(2\pi(5f_c)t) + \cdots$$

$$= \frac{1}{2}A_c m(t)\cos(2\pi f_c t)$$

$$+ \frac{2}{\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t)$$

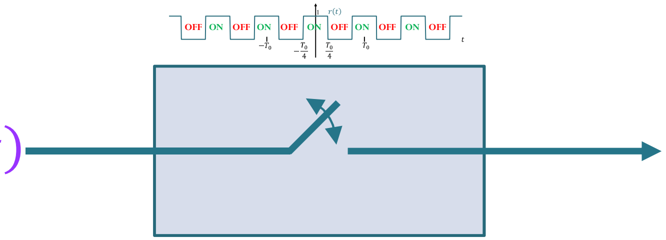
$$- \frac{2}{3\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi(3f_c)t)$$

$$+ \frac{2}{5\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi(5f_c)t) + \cdots$$

Now, recall that $\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$

Switching Demodulator

$y(t) = A_c m(t)\cos(2\pi f_c t)$

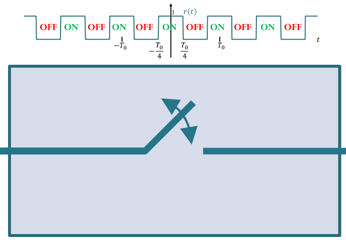


$y(t) \times r(t)$

$$
\begin{aligned}
y(t)r(t) = &\frac{1}{2}A_c m(t)\cos(2\pi f_c t) \\
&+\frac{1}{\pi}A_c m(t)\left(1+\cos\left(2\pi(2f_c)t\right)\right) \\
&-\frac{1}{3\pi}A_c m(t)\left(\cos\left(2\pi(f_c)t\right)+\cos\left(2\pi(5f_c)t\right)\right) \\
&+\frac{1}{5\pi}A_c m(t)\left(\cos\left(2\pi(3f_c)t\right)+\cos\left(2\pi(7f_c)t\right)\right)+\cdots
\end{aligned}
$$

Switching Demodulator

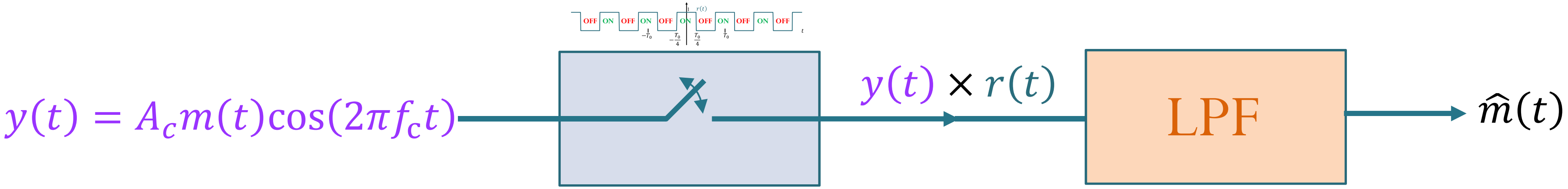
$y(t) = A_c m(t)\cos(2\pi f_c t)$



$y(t) \times r(t)$

$$y(t)r(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t)$$
$$+\frac{1}{\pi}A_c m(t) + \frac{1}{\pi}A_c m(t)\cos(2\pi(2f_c)t)$$
$$-\frac{1}{3\pi}A_c m(t)\cos(2\pi(2f_c)t) - \frac{1}{3\pi}A_c m(t)\cos(2\pi(4f_c)t)$$
$$+\frac{1}{5\pi}A_c m(t)\cos(2\pi(4f_c)t) + \frac{1}{5\pi}A_c m(t)\cos(2\pi(6f_c)t) + \cdots$$

Switching Demodulator



$$H_{\text{LPF}}(f)=\begin{cases} g, & |f|<B,\\ 0, & \text{otherwise.}\end{cases}$$

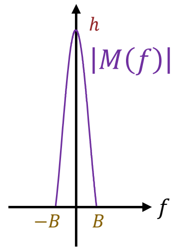
$$\begin{aligned} y(t)r(t) = &\frac{1}{2}A_c m(t)\cos(2\pi f_c t)\\ &+\frac{1}{\pi}A_c m(t)+\frac{1}{\pi}A_c m(t)\cos(2\pi(2f_c)t)\\ &-\frac{1}{3\pi}A_c m(t)\cos(2\pi(2f_c)t)-\frac{1}{3\pi}A_c m(t)\cos(2\pi(4f_c)t)\\ &+\frac{1}{5\pi}A_c m(t)\cos(2\pi(4f_c)t)+\frac{1}{5\pi}A_c m(t)\cos(2\pi(6f_c)t)+\cdots \end{aligned}$$

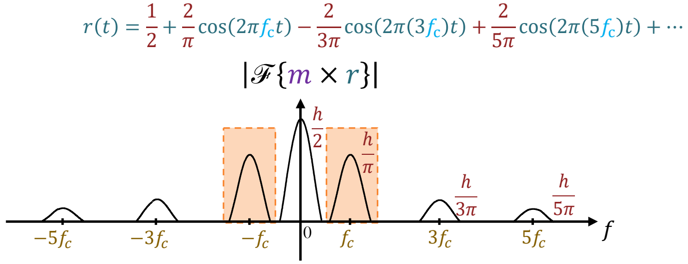
$$\hat{m}(t)=\frac{A_c g}{\pi}m(t)$$

$f_c > 2B$

Review: Switching Modem

Switching Modulator

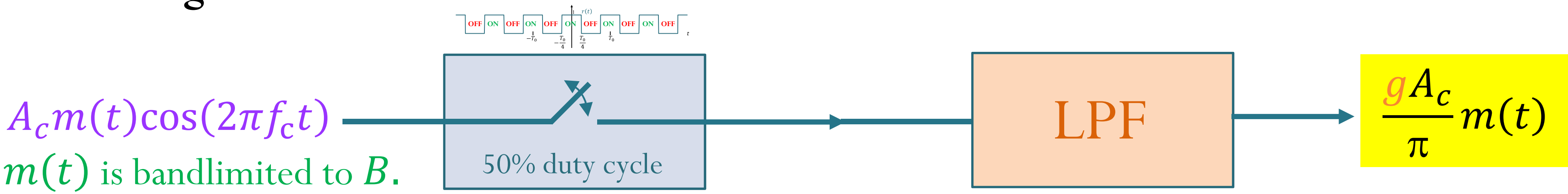




$f_c > 2B$

$$H_{\mathrm{BPF}}(f) = \begin{cases} g, & |f \pm f_c| < B, \\ 0, & \text{otherwise.} \end{cases}$$

Switching Demodulator



$m(t)$ is bandlimited to B.

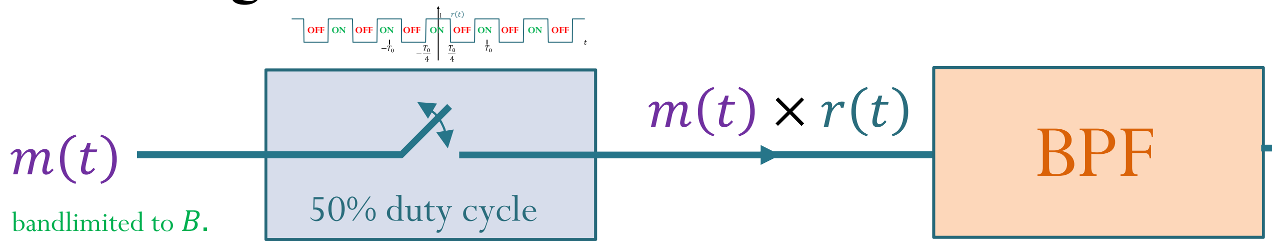
$f_c > 2B$

The ON time is synchronized with the nonnegative part of $\cos(2\pi f_c t)$.

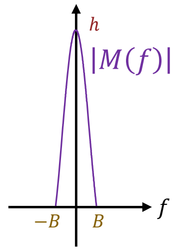
$$H_{\mathrm{LPF}}(f) = \begin{cases} g, & |f| < B, \\ 0, & \text{otherwise.} \end{cases}$$

Review: Switching Modem

Switching Modulator

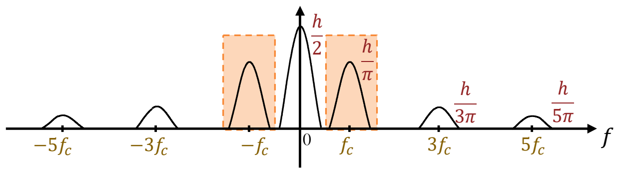


$$x(t) = \frac{2}{\pi} g m(t) \cos(2\pi f_c t)$$



$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$|\mathscr{F}\{m \times r\}|$

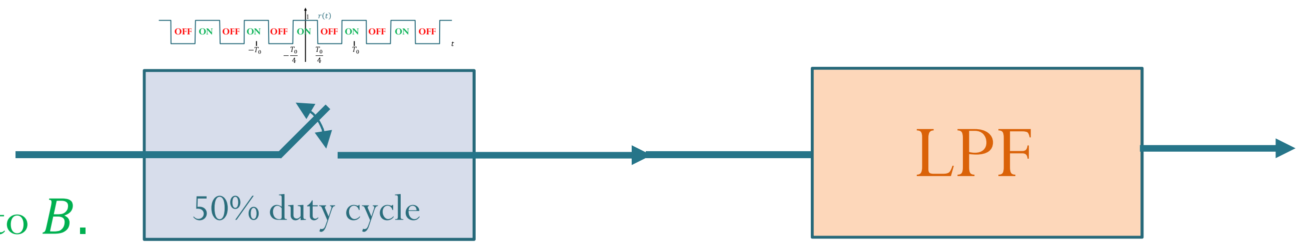


$f_c > 2B$

$$H_{\text{BPF}}(f) = \begin{cases} g, & |f \pm f_c| < W, \\ 0, & \text{otherwise.} \end{cases}$$

$B < W < f_c - B$

Switching Demodulator



$A_c m(t) \cos(2\pi f_c t)$

$m(t)$ is bandlimited to B.

$f_c > 2B$

$$\frac{g A_c}{\pi} m(t)$$

The ON time is synchronized with the nonnegative part of $\cos(2\pi f_c t)$.

$$H_{\text{LPF}}(f) = \begin{cases} g, & |f| < W, \\ 0, & \text{otherwise.} \end{cases}$$

$B < W < f_c - B$

Section 4.4

- **Crucial Skill 4.4.1**: Able to find/draw the output of the switching box both in time and frequency domains.
- Skill 4.4.2: Able to quickly find the output of the switching modulator and switching demodulator.